Chapter 8 / **Example 9**

Powers of complex numbers

a Given $\tan\left(\frac{\pi}{6}\right)=\frac{1}{\sqrt{3}}$ find the modulus and argument of $z=-\sqrt{3}-\mathrm{i}$.

b Find an expression for z^n and hence find the smallest value of n for which $\mathrm{Im}(z_n)=0$ and for this value of n give z^n in Cartesian form.

Open a new document and add a Calculator page.

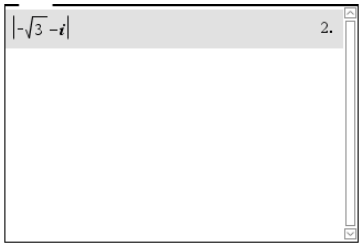
To enter the modulus function press [menu] and select [|□|] with the trackpad.

Type $-\sqrt{3}-\mathrm{i}$.

To enter i press [π] and select ***i*** from the menu.

Press [enter].

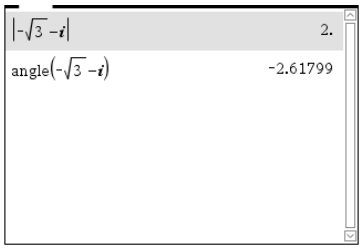
$\left|-\sqrt{3}-\mathrm{i}\right|=2$



Press [menu] 2:Number | 9:Complex Number Tools | 4:Polar Angle

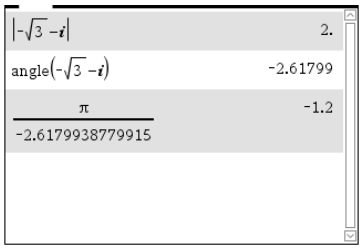
Type $-\sqrt{3}-\mathrm{i}$ and press [enter].

The argument is –2.62.



To find the argument in terms of π, type π [÷] [ctrl] [(−)] [ans] and press [enter].

The result is –1.2, so the argument is $\pi\div(-1.2)=-\frac{5\pi}{6}$.



Add a new Graphs page by pressing [ctrl] [doc] ([+page]).

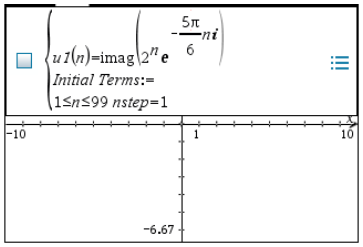
The entry line is displayed at the top of the work area.

Press [menu] 3:Graph Entry/Edit | 7:Sequence | 1:Sequence.

The default graph type is function, so '*u1*(*n*)=' is displayed.

The default axes are $-10\le x\le 10$ and $-6.67\le y\le 6.67$.

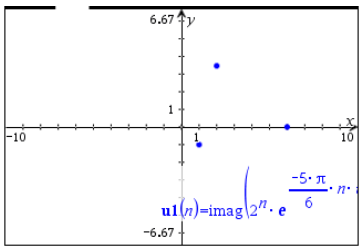
Type imag($2^n e^{-\frac{5n}{6}i}$, close the parentheses and press [enter] to enter the sequence as u1(*n*).



Chapter 8 / **Example 9**

Powers of complex numbers

The GDC displays a graph of the first few terms of the sequence.



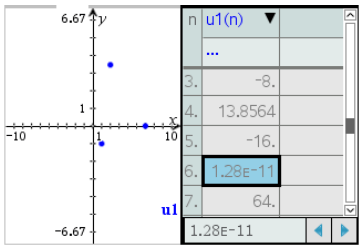
Press ctrl T.

A table of values is displayed alongside the graph.

Scroll down the table using ▼.

From the table, the smallest value of n for which $\text{imag}\left(2^n e^{-\frac{5n}{6}i}\right) = 0$ is 6.

Note that $1.28\text{E}-11 = 1.28\times10^{-11}$ *which is very close to zero. The difference is due to the numerical way in which the value is calculated.*



Return to the Calculator page by pressing ctrl ◀ del

Type $2^6 e^{-\frac{5n}{6}6i}$ and press enter.

$2^6 e^{-\frac{5n}{6}6i} = -64$.

Ignore $1.28\text{E}-11\cdot i$ *as it is close to zero.*